Turbo coded diversity system for mobile satellite communications

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Abstract—In this paper, the achievable performance of a turbo coded system adapted to a block fading channel model is evaluated. This model is suitable for analyzing, for instance, frequency-hopped multiple access (FHMA) systems that operate in a mobile satellite environment characterized by frequency-nonselective slow Rician fading, provided that the spacing between carriers is larger than the coherence bandwidth, resulting in basically uncorrelated blocks.

In such systems, coded information is transmitted over a small number of fading channels in order to achieve diversity. The best coded information allocations over a certain number of fading channels are evaluated. Bounds on the achievable performance due to coding are derived using information-theoretic techniques. Moreover, simulation results are obtained and compared with the theoretical ones.

I. INTRODUCTION

The block fading channel model [1] is motivated by the fact that in many wireless systems the coherence time of the channel is much longer than one symbol interval, resulting in adjacent symbols being affected by the same fading value. The fading blocks will experience independent fades, provided that we have sufficient separation in time, in frequency, or both in time and in frequency. An example of separation in frequency can be a frequency-hopped multiple access (FHMA) system that operate in a mobile satellite environment characterized by frequency-nonselective slow Rician fading, supposing that the spacing between carriers is larger than the coherence bandwidth, resulting in basically uncorrelated blocks [2]. An example of separation in time is a satellite-based time-division multiple access (TDMA) communication system, assuming that the TDMA frame guarantees a sufficient separation between the time-slots allocated to a single user [3]. An example of separation both in time and in frequency is a transponder (carrier) hopping TDMA system: there have been various implementations of such a system, i.e., the INTELSAT TDMA system or the TELECOM 1 system (see [4] and references therein).

Coding across different channels realizations provides a certain amount of diversity, counteracting the effects of multi-path fading. The most important advantage of such a system is that the amount of diversity is independent from the channel variation rate, since it is a result of exploiting frequency selectivity. In this work, the best coded information allocations across different fading channels are evaluated using information-theoretic techniques. Theoretical bounds on the achievable performance due to coding and simulation results are obtained and compared to assess the validity of the optimal allocation design procedure.

The paper is organized as follows. Section II provides a definition of the system model. Section III presents some observations on how to design the optimum interleaver to spread code symbols over the uncorrelated blocks. Section IV reports bound results on the achievable performance, derived using information-theoretic techniques, together with simulation results. Finally, Section V summarizes the main results and the conclusions.

II. SYSTEM MODEL

The model of the turbo encoded transmission system with diversity is shown in Fig. 1.

The information bits are encoded by a binary turbo code, consisting in the parallel concatenation of two equal rate-k/n systematic convolutional encoders and an interleaver. The k information sequences are transmitted together with the (n−k) check sequences generated by the second encoder; the (n−k) check sequences generated by the second encoder are also transmitted. The rate of the turbo code is then

\[ R = k/(2n-k) . \]

Information bits are coded by the parallel concatenated code into L blocks of length N symbols, being L the number of subchannels (or bursts in a TDMA system). These are denoted as

\[ x = (x_{1,1} \cdots x_{1,N} x_{2,1} \cdots x_{L,N}) . \]
We refer to these \( NL \)-dimensional codewords as frames. The coded symbols \( x \) formed by the turbo code are passed to an interleaver for practical reasons, the design of which will be discussed in the following section. In the analysis, we assume antipodal modulation, i.e., \( x_{i,j} = \pm 1 \), \( \forall i, \forall j \).

The complex envelope of the transmitted channel waveform \( s(t) \) for coded antipodal modulation can be expressed in the form:

\[
  g(t) = \sqrt{2E_s} \sum_{i} x_{i} g_0(t - iT) .
\]  

(2)

Here, \( E_s \) represents the energy per channel symbol, \( \{x_i\} \) is the binary sequence appearing at the output of a binary encoder and \( g_0(t) \) is the complex envelope of the transmitted channel signal with duration \( T \) and unit energy. When \( s(t) \) is transmitted over the channel characterised by Rician fading, in the hypothesis of multiplicative fading, a random amplitude \( a(t) \) and phase \( \Phi(t) \) are imposed onto \( s(t) \). The received signal contains a stable specular (direct) component and a random diffuse (multipath) component.

The received energy per channel symbol is the sum of the corresponding specular and diffuse energy. For convenience we impose the following normalisation:

\[
  E_s[\alpha^2] = \alpha^2 + 2a^2 = 1,
\]  

(3)

so that the received energy per channel symbol is \( E_s(\alpha) \). Note that \( \alpha^2 \) and \( 2a^2 \) are the normalised energy (with respect to \( E_s(\alpha) \)) of the specular and diffuse components, respectively. The channel parameter defined by \( K = \frac{\sigma_{ap}^2}{\sigma_s^2} \) represents the ratio of specular to diffuse energy (Rician factor). In terms of \( \alpha \) and \( a^2 \), the distribution of the amplitude process \( a(t) \) can be expressed as:

\[
  f(a) = \frac{a}{\sigma_a} \exp\left\{ -\frac{a^2 + a^2}{2\sigma_a^2} \right\} I_0\left( \frac{a \alpha}{\sigma_a} \right),
\]  

(4)

where \( I_0(\cdot) \) is the modified Bessel function of the first kind and zeroth order. The distribution \( f(a) \) is sufficiently general, since for Rayleigh fading \( K = 0 \), while if \( K \) approaches infinity the Rician channel reduces to the non-fading Gaussian channel (AWGN channel) with \( \alpha = 1 \).

Assume that the \( a(t) \) process varies slowly relative to an elementary signaling interval of \( T_s \) seconds, so that it can be considered constant over any such interval. The received signal is coherently demodulated under the assumption of perfect timing recovery and exact carrier phase tracking. The normalized matched filter output \( y_{i,j} \), corresponding to the symbol \( x_{i,j} \) transmitted on subchannel \( i \) at time \( j \), is given by [5]:

\[
  y_{i,j} = \sqrt{\frac{2E_s}{N_0}} a_{i,j} x_{i,j} + N_{i,j}, \quad i = 1, \ldots, L; \quad j = 1, \ldots, N.
\]  

(5)

Here, \( \{N_{i,j}\} \) is an independent identically distributed (IID) sequence of Gaussian variates with zero mean and unit variance. Moreover, the fading envelopes \( a_{i,j} \) of the \( L \) subchannels involved in each decoding process are assumed to be independent of each other, identically distributed, and constant over the subchannel. Namely, it is assumed that \( a_{i,j} = a_i \) \( (i = 1, 2, \ldots, L) \), \( \forall j \), i.e., on subchannel \( i \) the fading amplitude is assumed constant throughout the block sequence of length \( N \).

In addition to the decision variables \( y = \{y_{i,j}\} \), the decoder is supplied with \( a = \{a_{i,j}\} \), the channel amplitude estimates, from a channel estimator.

### III. Optimal Interleaver Design

The criterion for optimal interleaver design can be based on the minimization of an upper bound to the pairwise error probability (PEP) between two arbitrary codewords \( c_1 \) and \( c_2 \), as proposed in [6]. Owing to the code linearity, we will assume that the all-zero message has been transmitted. Define the fading envelope vector:

\[
  a = (a_1, a_2, \ldots, a_L),
\]  

(6)

where \( a_i \) \( (i = 1, 2, \ldots, L) \) represents the value of the envelope process on the \( i \)-th subchannel.

Assuming perfect phase tracking of the phase perturbation process and channel-state information at the receiver, the conditional pairwise error probability for an incorrect sequence with distance vector \( d = (d_1, d_2, \ldots, d_L) \) is [7]:

\[
  P(e_1 | d) = \frac{1}{2} \operatorname{erfc} \left( \sqrt{\frac{E_s}{N_0 L} \sum_{i=1}^{L} d_i a_i^2} \right),
\]  

(7)

where \( d_i \) is the Hamming distance between the portions of two codewords residing in block (subchannel) \( i \). The average error event probability can then be determined by averaging over the random \( L \)-vector \( a \) with the result:

\[
  P(e_1) = \frac{1}{2} E_a \left\{ \operatorname{erfc} \left( \sqrt{\frac{E_s}{N_0 L} \sum_{i=1}^{L} d_i a_i^2} \right) \right\},
\]  

(8)

where the expectation operator \( E_a \{\cdot\} \) represents joint expectation with respect to the components of \( a \).

Following the method used in [6], (8) can be upper bounded as:

\[
  P(e_1) \leq \frac{1}{2} \left( \frac{N_0}{\chi^2(d) E_s} \right)^{d_H} \exp \left( -\frac{K}{K+1} d_H \right),
\]  

(9)

where

\[
  \chi^2(d) = \left( \prod_{i=0}^{L} d_i \right)^{1/d_H},
\]  

(10)

and \( d_H \) is the number of nonzero \( d_i \)'s.

In order to find the best code symbol allocations on the different \( L \) subchannels, i.e., the best interleaver, the following procedure can be followed:

1. Fix a certain code symbol allocation on the different \( L \) subchannels.
TABLE I
ALLOCATION #1

<table>
<thead>
<tr>
<th>Bit type</th>
<th>Subchannel number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Information bits</td>
<td>1 1 1 1 1 1 1</td>
</tr>
<tr>
<td>First constituent check bits</td>
<td>2 2 2 2 2 2 2</td>
</tr>
<tr>
<td>Second constituent check bits</td>
<td>33 33 33 33</td>
</tr>
</tbody>
</table>

TABLE II
ALLOCATION #2

<table>
<thead>
<tr>
<th>Bit type</th>
<th>Subchannel number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Information bits</td>
<td>1 2 3 2 1 2 2 2</td>
</tr>
<tr>
<td>First constituent check bits</td>
<td>2 2 2 2 2 2 2 1</td>
</tr>
<tr>
<td>Second constituent check bits</td>
<td>3 3 3 3 3 3 2 2</td>
</tr>
</tbody>
</table>

TABLE III
ALLOCATION #3

<table>
<thead>
<tr>
<th>Bit type</th>
<th>Subchannel number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Information bits</td>
<td>1 2 3 2 1 2 1</td>
</tr>
<tr>
<td>First constituent check bits</td>
<td>2 1 3 2 1 2</td>
</tr>
<tr>
<td>Second constituent check bits</td>
<td>1 2 3 2 1 2</td>
</tr>
</tbody>
</table>

TABLE IV
ALLOCATION COMPARISON FOR L = 3

<table>
<thead>
<tr>
<th>Allocation #</th>
<th>E_n/N_0 [dB]</th>
<th>P_block</th>
<th>E_n/N_0 [dB]</th>
<th>P_block</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1.84e-02</td>
<td>2</td>
<td>4.25e-02</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>5.68e-02</td>
<td>2</td>
<td>2.85e-02</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>5.98e-02</td>
<td>2</td>
<td>3.02e-02</td>
</tr>
</tbody>
</table>

IV. RESULTS

A. Theoretical approach

The fading channel gain at time t can be denoted by \( a(t) \), with \( E[a^2] \) given by (3). Assume that the signal-to-noise ratio (SNR) is given by \( \Gamma(t) = a^2(t) \Gamma_{av} \), where \( \Gamma_{av} \) is the average SNR due to additive white Gaussian Noise (AWGN) present on the channel.

The approach followed in this work is to quantize the SNR range into \( Q + 1 \) intervals. The discrete-valued short-scale fading process \( \{c_k\} \) is, in this case, a memoryless process where the fading state \( \beta_k \) and the channel level \( c_k \) are the same [9].

As said previously, the interleaved code symbols are sent on the \( L \) uncorrelated subchannels in blocks of length \( N \).

The block error probability \( P_B \) of a binary code is lower bounded by [10]:

\[
P_B \leq 2^{-N E_p(R,s)},
\]

where \( E_p(R,s) = \max_{\rho \geq s} [E_0 - R \rho] \),

being

\[
E_0(\rho) = - \log_2 \int_{\rho^2}^{1} \frac{1}{\sqrt{\pi}} e^{-(s^2 + \frac{1}{\rho^2})} \left[ \cosh \left( \frac{2gs}{1 + \rho} \right) \right]^{1+\rho} \mathrm{d}s.
\]

The block error probability \( P_B \) of a code is also upper bounded by:

\[
P_B < 2^{-N E(R,s)},
\]

where

\[
E(R,s) = \max_{0 \leq s \leq 1} [E_0 - R \rho].
\]

Observe that, as long as the maximizing value of \( \rho \) is less or equal than 1, the two bounds are asymptotically equivalent and can be used to evaluate the best performance of a block code.

Actually, it has been shown [10] that turbo codes adopting spread interleavers perform within less than 1 dB of the sphere-packing bound, assuming \( P_B = 10^{-4} \). At higher error probabilities, the difference is even smaller.

On slow fading channels, it may be shown that the best performance (minimum average block error probability, \( E[P_B] \)) of a block code can be determined by:

\[
E[P_B] = \int_0^\infty P_B(\Gamma) f_{\Gamma}(\Gamma) \mathrm{d}\Gamma \approx \int_0^{\Gamma_0} f_{\Gamma}(\Gamma) \mathrm{d}\Gamma,
\]
where \( f_r (\Gamma) \) is the SNR probability density function, and \( \Gamma_0 \) is chosen so that \( P_D (\Gamma_0) = 0.5 \). Thus, the average best performance may be obtained by approximating the sphere-packing bound with a step function (the transmission is assumed to be error free if the signal to noise ratio satisfies the inequality \( \Gamma > \Gamma_0 \), and is assumed to be wrong otherwise). This approximation may be extended to the block fading channel as follows.

To determine if the total transmission of the \( NL \)-dimensional codeword (i.e., frame) may be assumed to be successful or not, given a certain SNR distribution \( s \) over the \( L \) subchannels, introduce the relevant code rate, \( R_i \), of the \( i \)-th block, \( (i = 1, 2, ..., L) \), that is calculated from (12) and (15) Vi as the rate needed to achieve the target block error probability, \( P_B = P_s (P_o) \). Define sustainable rate, \( R_s \), as the weighted average value of the relevant code rates, corresponding to the transmissions of the different blocks.

Given that all blocks include the same amount of channel bits, here \( R_s \) is calculated as:

\[
R_s = \frac{\sum_{i=1}^{L} R_i}{L}. \tag{18}
\]

The total transmission of the \( NL \)-dimensional codeword (i.e., frame) is assumed to be successful if the inequality \( R_s > R_s \) is satisfied, i.e., if the rate of the code actually used is lower than the sustainable rate, and is assumed to be unsuccessful otherwise. The residual FER at the decoder output, conditioned on a fixed SNR distribution \( s \) over the \( L \) subchannels \( (\alpha_1^2, \Gamma_1, \alpha_2^2, \Gamma_2, ..., \alpha_L^2, \Gamma_L) \), can thus be determined as:

\[
\text{FER}(\alpha_1^2, \Gamma_1, \alpha_2^2, \Gamma_2, ..., \alpha_L^2, \Gamma_L) = \Pr \left[ R_s < R_s \left| \alpha_1^2, \Gamma_1, \alpha_2^2, \Gamma_2, ..., \alpha_L^2, \Gamma_L \right. \right]. \tag{19}
\]

The number of possible SNR distributions is \( D = Q^L \) being \( Q \) the possible fading states \( \beta_k \). Thus, the average FER is given by:

\[
\text{FER}(\Gamma_1, \alpha_1^2, \alpha_2^2, ..., \alpha_L^2) = \sum_{\alpha_1^2, \alpha_2^2, ..., \alpha_L^2} \text{FER}(\alpha_1^2, \Gamma_1, \alpha_2^2, \Gamma_2, ..., \alpha_L^2, \Gamma_L) \cdot \Pr[\alpha_1^2] \cdot \Pr[\alpha_2^2] \cdot ... \cdot \Pr[\alpha_L^2]. \tag{20}
\]

being, given the memoryless assumption

\[
\Pr[\alpha_i^2] = \left( \frac{\alpha_i^2}{\sum_{j=1}^{L} \alpha_j^2} \right)^2. \tag{21}
\]

In Fig. 2 the theoretical FER values (solid curves) are reported as a function of the average signal-to-noise ratio \( \Gamma \) for different values of the number of subchannels \( L \). The curve with the label \( L = 1 \) is obtained for an ideal slow multipath fading channel. For small values of \( L \) (i.e., \( L \leq 3 \)) (20) can be evaluated directly, whereas for greater \( L \) (i.e., \( L \geq 4 \)) one can resort to Monte Carlo evaluation.

**B. Simulation results**

In Fig. 2, are also reported the residual FER values versus \( E_b / N_0 \) obtained by simulation (dotted curves). For each value of \( L \) (except \( L = 3 \) the curves are obtained having assumed to use the best interleaver, following the design rules described in Section III. In Tables V, VI and VII are reported the optimal code symbol allocations for \( L = 2, L = 4 \) and \( L = 10 \), respectively.

For \( L = 3 \), the dotted curve reports the simulation values obtained with the code symbol allocation reported in Table I (non optimal), whereas the dashed curve reports the simulation values obtained with the optimal code symbol allocation reported in Table II. The dotted curve on the left side reports the values obtained \( L = \infty \), i.e., for an ideal fast multipath fading channel: in this case, the fading amplitude is assumed to be independent from symbol to symbol all over the frame (i.e., \( N = 1 \)).

In Figs. 3 and 4, are reported the residual BER values and throughput values, respectively, obtained by simulation. The

**TABLE V**

**OPTIMAL ALLOCATION WITH \( L = 2 \)**

<table>
<thead>
<tr>
<th>Bit type</th>
<th>Subchannel number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Information bits</td>
<td>1 2 1 2 1 2 1 2 1 2 ...</td>
</tr>
<tr>
<td>First constituent check bits</td>
<td>1 2 1 2 1 2 1 2 1 2 ...</td>
</tr>
<tr>
<td>Second constituent check bits</td>
<td>1 2 1 2 1 2 1 2 1 2 ...</td>
</tr>
</tbody>
</table>

**TABLE VI**

**OPTIMAL ALLOCATION WITH \( L = 4 \)**

<table>
<thead>
<tr>
<th>Bit type</th>
<th>Subchannel number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Information bits</td>
<td>1 3 3 1 3 3 1 3 3 ...</td>
</tr>
<tr>
<td>First constituent check bits</td>
<td>2 4 2 4 2 4 2 4 2 4 ...</td>
</tr>
<tr>
<td>Second constituent check bits</td>
<td>3 2 4 2 4 2 4 2 4 2 ...</td>
</tr>
</tbody>
</table>
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