Analysis and Design of Interleavers for CDMA Systems

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Abstract — In the last years, the Turbo-principle has been applied to multiuser receivers in CDMA systems since [1]. However, the role of interleavers in these Turbo-receivers has not been studied yet. Our purpose is to partially fill this gap.

I. SYSTEM DESCRIPTION

Let us consider K users transmitting on the same AWGN channel. Each user encodes its message d to the same code C and obtains the encoded message b. The sequence is BPSK mapped, spread and sent to the channel. At the receiver, a user separator (US) performs some filtering and outputs K streams, each entering a single-user channel decoder. The latter performs decoding through either a Viterbi or a BCJR algorithm.

II. C-OPTIMAL INTERLEAVERS

Let C be a linear block code with parameters (n, k). The interleaver I is a permutation of the integers 1, ..., n. Let I(C) be the equivalent code obtained by permuting I with all the codewords in C and define C' = C \cap \Pi(C). It is easy to prove that C' is a subcode of C. The interleaver I is called C-optimal if and only if

\[ \dim(C') = k - \min\{k, n-k\}. \]  

The rationale of this definition lies in the operation the decoders perform. In fact, they accept the input as if coming from a single-user memoryless channel, while the other users' coded signals give rise to an interference with memory. The only way to destroy this memory is a good interleaving. Suppose k > n - k. If I is C-optimal, the fraction of code words in C that are also in I(C) is \( \frac{2^{k-n}}{2^k} \) by the definition. But this is also the probability of belonging to C', for an n-dimensional word output by a binary memoryless source. If k \leq n - k, then C' contains only the all-zero code word, which is obviously resistant to all permutations.

Codes with k \leq n - k that possess the all-one code word do not have optimal permutations, because the all-one code word belongs to C' for every choice of I.

Denote with \Pi_j the j-th user's interleaver. The K interleavers \Pi_1, ..., \Pi_K are said to be mutually C-optimal if, for every choice of i and j, \Pi_i,\Pi_j^{-1} is C-optimal.

III. A SET OF MUTUALLY C-OPTIMAL INTERLEAVERS

Since there are serious difficulties for a general theory, we limit our analysis to the particular set of congruential interleavers [2], defined by the following:

\[ \Pi_g(i) = ig \mod n. \]  

We consider an \((n_0, k_0 = 1, N)\) convolutional code, terminated after a block of L trellis steps, corresponding to \( n_0L \) coded bits. Denote by \( N' \) the constraint length of its dual code. Limiting the analysis to symbol interleavers, which are permutations of the integers 1, ..., L, the following theorem holds:

**Theorem 1** Consider a block length L such that

\[ 1 + Z + \ldots + Z^{L-1} = \frac{1 + Z^L}{1 + Z} \]  

is an irreducible polynomial over GF(2). If \( L > \frac{N'N'}{k_0(N_0-k_0)} \), then \( \Pi_g \) is C-optimal for 1 \( \leq g < L - 1 \).

In Figure 1 we show the effect of interleaving in a synchronous CDMA scenario with four equal-power users and the iterative receiver introduced in [3]. The solid lines correspond to optimal interleaving while the dashed lines refer to absence of interleaving.

![Figure 1: Effect of interleaving. The US is a linear MMSE filter until the third iteration, then it is a conventional one.](image)

**REFERENCES**

