A Linear Front End for Iterative Soft Interference Cancellation and Decoding in Coded CDMA

A. Tarable, G. Montorsi, S. Benedetto
Politecnico di Torino
C.so Duca degli Abruzzi 24, Torino (Italy)
E-mail: {tarable, montorsi, benedetto}@polito.it

Abstract—A new suboptimal Turbo receiver for coded CDMA is presented. The user-separating front-end of this receiver has been thought of as the extension of linear multiuser detectors to an iterative structure. The complexity of this receiver is shown to be \( O(K) \), where \( K \) is the number of users. Simulations show that, for sufficiently high signal-to-noise ratios, the linear-US receiver tends to single-user performance.

I. INTRODUCTION

In the past fifteen years, code-division multiple access (CDMA) has been a subject of great interest, because of its attractive properties. Since the multiaccess channel is equivalent to a code, many authors thought of a Turbo receiver ([1]-[5]), in which, at every iteration, a user-separating soft-input soft-output block (called hereafter US-SISO) exchanges information with each user's SISO decoder to improve performance. Like in uncoded CDMA, the optimal US-SISO is exponentially complex with the number of users [3], so it is important to think of a more feasible realization. Among the others, Wang and Poor [3] proposed a US-SISO, described later in the article, which is a natural extension of a nonlinear multiuser receiver. In this paper, we follow the ideas in [3] and design a linear US-SISO, which requires a linear transformation only before the first iteration, to obtain a lower-complexity Turbo receiver. We will show that, although its performance is slightly worse, our receiver approaches the single-user limit for sufficiently high signal-to-noise (SNR) ratios.

II. SYSTEM DESCRIPTION

We consider, for simplicity, \( K \) synchronous users on an AWGN channel; the \( j \)-th user's information bit stream \( d_j = \{d_j[1], ..., d_j[M]\} \in \{-1, 1\}^M \), where \( M \) is the frame length, is passed to the encoder; the corresponding coded bit stream, after the interleaver, will be denoted \( b_j = \{b_j[1], ..., b_j[L]\} \in \{-1, 1\}^L \), where \( L = M/R_c, R_c \) being the code rate. In the \( i \)-th symbol interval, \( i = 1, ..., L \), with duration \( T \), the received signal will be

\[
y[i](t) = \sum_{j=1}^{K} A_j b_j[i] s_j(t - (i - 1)T) + n(t),
\]

where \( A_j \) is the \( j \)-th user amplitude, \( s_j(t) \) is his (short) spreading waveform, with support \([0, T]\) (all are supposed to be known at the receiver), and \( n(t) \) is Gaussian noise, with zero mean and variance \( \sigma^2 \). See Figure 1 for a block diagram of the transmitter. In the receiver, after the matched filter bank, we have, for each symbol index \( i \), a vector with \( K \) elements

\[
y[i] = R A b[i] + n[i],
\]

\( R \) being the \( K \times K \) correlation matrix between the users, with elements \( R_{ij} = \int_0^T s_j(t) s_i(t) \, dt \), \( A \) the \( K \times K \) diagonal matrix of amplitudes, \( b[i] = \{b_1[i], b_2[i], ..., b_K[i]\}^T \) the \( K \)-dimensional vector of interfering bits and \( n[i] \) a vector of noise samples, with zero mean and covariance matrix \( E[n[i]n^T[i]] = \sigma^2 R \). The vector \( y[i] \) is a sufficient statistic for detecting \( b[i] \), that is, we do not incur any loss of information in passing from \( y[i](t) \) to \( y[i] \) [6].

Since every bit interval is independent from the others, we may as well drop the index \( i \), unless we expressly need it.

III. DESCRIPTION OF THE TURBO RECEIVER

The cascade (users' encoders + interleavers + CDMA channel) can be viewed as a serial concatenation of encoders, with the CDMA channel acting as the inner encoder. It is straightforward to extend to this system the iterative decoding [7]. The conceptual scheme of the Turbo receiver, in a synchronous CDMA scenario, can be briefly outlined by a step-by-step al-
Figure 2 – Turbo receiver. At every iteration, the US-SISO exchanges information with the decoders; IL $^{-1} =$ deinterleaver.

Algorithm in the following manner (see Fig.2 for a picture of the iterative receiver):

1. $\text{ExInf}_2(k,i)$ is set to 0, for all $k$, $i$;

2. From the received vector $y[i]$, and the current bit statistics $\text{ExInf}_2(k,i)$, the US-SISO computes the log-likelihood ratio $\text{LLR}_1(k,i) =$ \log $\frac{\Pr(y_k|b_k = 1)}{\Pr(y_k|b_k = -1)}$, $\forall k$, $\forall i$, and extracts the extrinsic information $\text{ExInf}_1(k,i) = \text{LLR}_1(k,i) - \text{ExInf}_2(k,i)$; Now in parallel for each of the $K$ users:

3. The vector of $\text{ExInf}_1(k,i)$'s is deinterleaved to obtain the input to the $k$-th decoder, $\text{ExInf}_1(k)$;

4. The $k$-th decoder computes the $\text{LLR}_2(k,i)$'s;

5. The vector of $\text{LLR}_2(k,i)$'s is again interleaved to obtain $\text{LLR}_2(k,i) =$ \log $\frac{\Pr(y_k|b_k = 1)}{\Pr(y_k|b_k = -1)}$, $\forall i$ and the extrinsic information is extracted: $\text{ExInf}_1(k,i) = \text{LLR}_2(k,i) - \text{ExInf}_1(k)$;

6. If this is not the last iteration, the algorithm returns to step 2, otherwise it stops.

In the last iteration each decoder must also supply the final estimate of the corresponding information bit stream, which represents the output of the whole receiver.

Wang and Poor's low-complexity US-SISO [3] first subtracts from $y$ a soft estimate of the MUI term, that is, it computes $y_k = y - RA\hat{b}_k$, where

$$\hat{b}_k = \begin{cases} \Pr[b_j = 1] - 1, & j \neq k \\ 0, & j = k \end{cases}$$

Then, it performs a linear MMSE processing on $y_k$, namely, it obtains $z_k = w_k^T y_k$, where $w_k$ is the solution to the standard MMSE problem:

$$w_k = \arg \min_{w \in \mathbb{R}^K} E \left[ (b_k - w^T y_k)^2 \right].$$

The extrinsic information is extracted from $z_k$. Since $z_k$ is nearly Gaussian [8], this can be done in a straightforward way. The receiver in [3] requires solving the MMSE problem (4) at each iteration, and this means inverting a $K \times K$ matrix, so its complexity is about $O(K^3)$ operations per decoded bit per iteration per user.

IV. THE LINEAR US-SISO

Before the first iteration, the linear US-SISO performs an MMSE transformation of the channel output $\tilde{y} = My$, where $M = (R + \sigma A^{-2})^{-1}$. Afterwards, for the first $m$ iterations, it extracts from $\tilde{y}$ the extrinsic information, on the basis of the bit statistics at that iteration. The expression of $\tilde{y}_k$, normalized to have unit noise variance, is the following:

$$\tilde{y}_k = a_{kk} b_k + \sum_{j \neq k} a_{kj} b_j + \tilde{n}_k,$$

where $a_{kj} = \frac{(MR)_{kj} / \sqrt{(MRM)}_{jj}}{\sqrt{(MRM)}}$, are coefficients that take into account the amplitudes and correlations between users, and $\tilde{n}_k \sim \mathcal{N}(0, 1)$. Making a Gaussian approximation of the MUI + noise term, we can write:

$$\tilde{y}_k \simeq a_{kk} b_k + \tilde{\nu}_k,$$

where $\tilde{\nu}_k \sim \mathcal{N}(\tilde{\mu}_k, \tilde{\sigma}^2_k)$, $\tilde{\mu}_k$ and $\tilde{\sigma}^2_k$ being respectively the mean and the variance of the MUI + noise term. Having, at a given iteration, for the $j$-th user $\Pr\{b_j = 1\} = p_j$, and calling $\tilde{b}_j = E[b_j] = 2p_j - 1$, a straightforward computation shows that:

$$\tilde{\mu}_k = \sum_{j \neq k} a_{kj} \tilde{b}_j,$$

$$\tilde{\sigma}^2_k = 1 + \sum_{j \neq k} a_{kj}^2 \left( 1 - \tilde{b}_j^2 \right).$$

Then, one can simply extract the LLRs in a direct way, that is:

$$\text{LLR}_k = \frac{\Pr\{\tilde{y}_k | b_k = 1\}}{\Pr\{\tilde{y}_k | b_k = -1\}} = \frac{2a_{kk}}{\tilde{\sigma}^2_k} (\tilde{y}_k - \tilde{\mu}_k),$$

which makes evident the fact that the linear US-SISO makes a soft interferers' cancellation. Fig. 3 shows the structure of the linear US-SISO. The reliability of the extrinsic information is determined by $\tilde{\sigma}^2_k$, which decreases when the interferer bit estimates approach $\pm 1$. It is worth noting that also a wrong statistic contributes to reduce the variance, though it really raises interference.

It is known that the MMSE receiver is near-far resistant, and that implies that also the linear-US receiver is near-far resistant. In fact, in the ideal case in which the interferers' amplitudes tend to infinity, the MMSE US-SISO acts like a decorrelating receiver, and at the first iteration it provides a full MUI cancellation.
However, the MMSE US-SISO does not tend to the single-user limit, even if the interferers are completely cancelled. In fact, to measure the limit performance of the MMSE US-SISO, we can find the (hard) error probability in the ideal case in which a genius reveals the interferers’ bits. In that case, $b_j = b_j$, for all $j$, and, from Eqs. (7), (8) and (9), the following error probability is obtained:

$$P_o(e_k) = \frac{Q(\alpha_{hk})}{2},$$

(10)

which can be found to be always worse than the single-user error probability and to approach the latter only when $A_j \to 0$, $\forall j \neq k$.

So, after the $m$-th iteration, when the bit statistic is sufficiently reliable, the MMSE US-SISO switches to a conventional US-SISO: that is, it performs the same operations as before, but on the matched filter output, instead of the MMSE-filter output. Explicitly, Eqn. (9) becomes:

$$\text{LLR}_k = \frac{\Pr\{y_k | b_k = +1\}}{\Pr\{y_k | b_k = -1\}} = \frac{2A_k}{\sigma_k^2} (y_k - \mu_k),$$

(11)

where:

$$y_k = A_k b_k + \sum_{j \neq k} \rho_{kj} A_j b_j + n_k,$$

(12)

$$y_k \approx A_k b_k + v_k,$$

(13)

being $v_k \sim N(\mu_k, \sigma_k^2)$, with:

$$\mu_k = \sum_{j \neq k} \rho_{kj} A_j b_j,$$

(14)

$$\sigma_k^2 = 1 + \sum_{j \neq k} \rho_{kj}^2 A_j^2 \left(1 - \frac{1}{2}\right).$$

(15)

Unlike the MMSE US-SISO, the conventional US-SISO grants a single-user error probability in the ideal genius-aided case. This is the reason why the conversion at the $m$-th iteration is necessary. The optimization of the iteration $m$ at which the receiver switches is still an open problem.

Let us consider the complexity of the new receiver. The linear US-SISO performs only the first time the linear filtering, and in the successive iterations it subtracts the current MUI estimation. This task has a complexity which grows linearly with the number of users, that is, it needs about $O(K)$ operations per decoded bit per iteration per user. It is thus simpler than the receiver proposed in [3], which needs a different linear transformation at each iteration.

Conceptually, the receiver in [9] performs the same operations, but on the matched filter outputs from the beginning (that is, $m = 0$ for that receiver). However, as we verified, it does not work when strong interferers are present. That is, it still suffers the near-far problem, although the iterative structure improves its behaviour and makes the near-far conditions for that receiver less stringent than for a plain conventional receiver.

V. NUMERICAL ANALYSIS

We exactly performed the same simulations as [3], to permit direct comparison of the results. We consider four users and use for each user the same convolutional code with rate 1/2, memory $\nu = 4$ and generators 23, 35 (in octal form). The interleavers are different for each user and are random. The SISO decoders are based on the BCJR algorithm [10]. The correlations are 0.7 for all users. Initially we have supposed perfect power control, so all users have the same power. For the US-SISO switches, we chose $m = 2$ (in the first two iterations, the US-SISO is MMSE). The results are shown in Fig. 4. There is a loss of performance between our receiver and Wang and Poor’s because of the reduced complexity. However, the curves show that our receiver tends to the single-user limit for sufficiently high SNR.

Another simulation has been performed, again with four users, which considers two users 3dB stronger than the other two. Otherwise the simulation is identical to the previous one. In this case we can observe that the receiver still performs well enough and converges to the single-users curve. In Figs. 5-6 the performance for both the weak and the strong users are depicted. It can be noticed that the strong users suffer from the presence of the weak users. Their convergence to the single-user curve seems to be anchored to the weak users’ amplitude. This behaviour has been observed also in [3].

We also analyzed the performance of our receiver with the technique introduced by Ten Brink [11]. We consider a symmetric channel, i.e. a channel in which the users have equal powers and equal correlation between each other. We can define a mutual information between the $k$-th user’ $j$-th coded bit and the corresponding ExInf output by the US-SISO at the $l$-th iteration, which we denote $I(l)$ (ideally not dependent from $k$ and $j$ because of the symmetry). Our purpose is to plot the iteration gain $I(l+1) = f[I(l)]$ for the linear-US receiver.

In Figs. 7-8, the curves for the linear-US receiver and, comparatively, for the receiver in [3] (parametrized with the users’ amplitude) are plotted, together with the same curves reflected by symmetry with respect to the I-III quadrant diagonal. In

Figure 3 – Linear US-SISO. The switch commutes after the $m$-th iteration.
In this way, the evolution of mutual information along the iterations can be easily derived, following a zig-zag path starting from the origin, as shown in Fig. 8. Practically, we opened the loop after the US-SISO and injected as input ExInfs a vector of independent Gaussian variables. The receiver worked for an iteration and we measured the mutual information on the ExInfs output by the US-SISO after that iteration.

In Figs. 7-8, the amplitudes are $A = 1$ and $A = 2$ respectively, and the noise variance is $\sigma^2 = 1$. There are four users and the correlation between any pair of users is 0.7. For our receiver, the first iterations are on the MMSE curve, then, when the US-SISO switches, the path must be followed on the conventional curve. It can be seen that, for $A = 1$, none of the two receivers converges to the single-user limit (the asymptotic value of the conventional curve for $I(i) \to 1$), and that switching to conventional actually worsen the behaviour of our receiver, because it converges to the intersection point between the conventional curve and its reflected, which is lower than the corresponding intersection point for the MMSE curve. Instead, for $A = 2$ both receivers converge to single-user performance. From the curves, the Wang-Poor receiver appears to be a compromise between the MMSE and the conventional, like the former for low input information (denoted $I(i)$ in the Figures), and like the latter for high $I(i)$.

From the EXIT charts, it would seem that also with $m = 0$ (always conventional), the receiver seems to work. However, the hypothesis that the input ExInfs were uncorrelated is not true, as the iterations proceed, and this discrepancy is more disturbing for the conventional curve, and for low input information. In that case, the EXIT charts do not predict correctly the behaviour of the receiver. For further details, see [12].

In this paper, we have proposed a new low-complexity Turbo receiver for a coded CDMA system. The linear-US receiver performs a linear MMSE transformation before the first iteration. Then, at the first iterations, it extracts from the MMSE filter outputs the ExInfs to be sent to the SISO decoders, on the basis of the current bit statistics. When the bit estimates are sufficiently reliable, it directly uses the matched filter outputs to obtain the ExInfs.

Simulations showed that the linear-US receiver tends to the single-user limit for sufficiently high SNRs. Its performance loss, if compared with Wang-Poor receiver, is compensated by its very low complexity, linear in the number of users.

VI. CONCLUSIONS

REFERENCES


Figure 6 – Results of simulation. The system is the same as Figure 5. Performance of the strong users.

Figure 7 – Exit chart for the linear-US receiver. The system is the same as Figure 5. $A = 1$.

Figure 8 – Exit chart for the linear-US receiver. The system is the same as Figure 5. $A = 2$.


